6. D. C. Courzens and D. H. Trevena, "Critical tension in a liquid under dynamic conditions of stressing," Nature, 222, No. 5192 (1969).
7. A. N. Dremin, G. I. Kanel', and S. A. Koldunov, "Study of cleavage in water, ethyl alcohol, and Plexiglas," in: Combustion and Explosion. Materials of the III All-Union Symposium on Combustion and Explosion [in Russian], Nauka, Moscow (1972).
8. G. A. Carlson and K. W. Henry, "Technique for studying tensile failure in application to glycerol," J. Appl. Phys., 44, No. 5 (1973).
9. V. K. Kedrinskii, "Dynamics of the cavitation zone in an underwater explosion near a free surface," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1975).
10. S. V. Stebnovskii, "Development of initial perturbations of the external boundary of an expanding gas–liquid ring," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1982).
11. K. A. Naugol'nykh and N. A. Roi, Electrical Discharges in Water [in Russian], Nauka, Moscow (1971).

STATIONARY FLOWS IN CHANNELS DURING SIMILARITY WAVE PROPAGATION OF A CHEMICAL REACTION WITH AN ABRUPT CHANGE IN VISCOSITY

G. V. Zhizhin and A. S. Segal

UDC 532.542:660.095.26

The channel flow of reacting media with a viscosity which increases sharply with the degree of chemical conversion is accompanied by adhesion of the product to the channel wall and the formation of a stream of unreacted substance along the axis [1-6]. This phenomenon was studied numerically in [2, 3] and experimentally in [4] using the example of a polymerization in the continuous tube reactor. An analytic description was given in [5, 6] of the dynamics of the process with the condition that the viscosity and density of the medium is unambiguously dependent on the time of the reaction. This condition is satisfied in the absence of heat transfer in the medium (for example, in the isothermal flow of a medium with negligible diffusion). However, many chemical reactions are accompanied by intensive heat release and heat transfer, and the results obtained in [5, 6] are inapplicable in these cases. This pertains particularly to the practically important phenomenon of similarity wave propagation of a reaction, which was examined in [7-11] for the case of polymerization in a stationary medium. To allow for molecular heat transfer – which is important in this case – the method developed in [5, 6] was modified in [12]. However, the authors could not obtain an analytic description of flow in this case and instead presented results of numerical calculations of some of its characteristics.

Here we examine stationary flows of a reacting medium in plane and cylindrical channels during the similarity wave propagation of the reaction accompanied by a sharp increase in viscosity. The study will be based on the method of calculating laminar flows in a boundary layer with a surface of discontinuity. This method was developed in [13].

Within the framework of the assumptions made in [12], we obtained an analytic solution to the problem and found the form of the reaction wave in explicit form. The flow-rate – pressure characteristics of the channel were determined. It is shown that a small parameter of the problem – the ratio of the viscosities of the medium before and after the wave – introduces a singular perturbation into the solution which results in a sharp distortion and extension of the wave profile and the profile of the longitudinal velocity component near the axis, i.e., it leads to the formation of an axial jet. As a consequence of this, the condition of uniform smallness of the angle of inclination of the wave to the axis that was adopted in [12] is not satisfied, and the stationary reaction wave ceases to exist away from the axis. The region of existence of the wave becomes smaller with a decrease in the ratio of viscosities on the wave, which may lead to destabilization of the steady-state process.

1. We are examining stationary flows in plane and cylindrical channels in the presence of a chemical reaction wave. The medium is assumed to be incompressible, and heat release due to dissipation of mechanical energy is not considered. The thickness of the wave is assumed to be small in comparison to the characteristic cross-sectional dimension of the channel.

Leningrad. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 61-68, January-February, 1986. Original article submitted October 5, 1984.

 0021-8944/86/2701-0054$12.50 © 1986 Plenum Publishing Corporation

Given these assumptions, the wave can be regarded as a surface of discontinuity. Meanwhile, the projection of the local velocity of the flow on the normal to this surface is equal to the velocity of a wave in a stationary medium (the local law of Michelson [14]).

As is known [14], stable stationary chemical-reaction waves in channels are observed at mean-flow-rate velocities $U(\varepsilon = V/U \ll 1)$ which are high compared to V. Thus, by virtue of the local Michelson law, the transverse components of flow velocity are on the order of V, while the longitudinal components are obviously on the order of U.

Let us examine a flow in a plane channel. We introduce a Cartesian coordinate system (x, y) so that its x axis coincides with the channel axis. The scales of the hydrodynamic quantities are chosen so that the corresponding dimensionless variables will be on the order of unity [13, 15]: V is the scale of the transverse component of velocity v; U is the scale of the longitudinal component of velocity u, the distance from the channel axis to the wall of the channel; Y is the scale of the coordinate y; $X = YU/V$ is the scale of the coordinate x; $P = \mu_2 U^2/YV$ is the scale of the pressure gradient (μ_2 is the viscosity of the reaction product). Then referring all of the quantities to their scales, we write the equations of motion in dimensionless variables:

$$\mathrm{Re}_2\left(u_1\frac{\partial u_1}{\partial x}+v_1\frac{\partial u_1}{\partial y}\right)=-\frac{\partial p_1}{\partial x}+\alpha\left(\varepsilon^2\frac{\partial^2 u_1}{\partial x^2}+\frac{\partial^2 u_1}{\partial y^2}\right); \tag{1.1}$$

$$\varepsilon^2\mathrm{Re}_2\left(u_1\frac{\partial v_1}{\partial x}+v_1\frac{\partial v_1}{\partial y}\right)=-\frac{\partial p_1}{\partial y}+\alpha\left(\varepsilon^4\frac{\partial^2 v_1}{\partial x^2}+\varepsilon^2\frac{\partial^2 v_1}{\partial y^2}\right); \tag{1.2}$$

$$\frac{\partial u_1}{\partial x}+\frac{\partial v_1}{\partial y}=0; \tag{1.3}$$

$$\mathrm{Re}_2\left(u_2\frac{\partial u_2}{\partial x}+v_2\frac{\partial u_2}{\partial y}\right)=-\frac{\partial p_2}{\partial x}+\varepsilon^2\frac{\partial^2 u_2}{\partial x^2}+\frac{\partial^2 u_2}{\partial y^2}; \tag{1.4}$$

$$\varepsilon^2\mathrm{Re}_2\left(u_2\frac{\partial v_2}{\partial x}+v_2\frac{\partial v_2}{\partial y}\right)=-\frac{\partial p_2}{\partial y}+\varepsilon^4\frac{\partial^2 v_2}{\partial x^2}+\varepsilon^2\frac{\partial^2 v_2}{\partial y^2}; \tag{1.5}$$

$$\frac{\partial u_2}{\partial x}+\frac{\partial v_2}{\partial y}=0, \tag{1.6}$$

where $\alpha = \mu_1/\mu_2$; $\mathrm{Re}_2 = VY/\mu_2$ is the Reynolds number; all of the dimensionless variables have the same notation as their dimensional analogs; the subscript 1 pertains to flow up to the surface of discontinuity; the subscript 2 pertains to flow after this surface.

Following [12, 13], we assume that the angle of inclination of the surface of discontinuity to the channel axis β is uniformly small over the entire flow region and is on the order of ε. Then the conditions of conservation of mass and the normal and tangential components of momentum on the surface of discontinuity in dimensionless variables take the form

$$(u_1\mathrm{tg}\beta - v_1)\,|_F = (u_2\mathrm{tg}\beta - v_2)\,|_F; \tag{1.7}$$

$$\left[\alpha\left(2\varepsilon^2\frac{\partial u_1}{\partial x}\mathrm{tg}\,\beta-\frac{\partial u_1}{\partial y}-\varepsilon^2\frac{\partial v_1}{\partial x}\right)-p_1\,\mathrm{tg}\,\beta\right]\Big|_F=\left(2\varepsilon^2\frac{\partial u_2}{\partial x}\,\mathrm{tg}\,\beta-\frac{\partial u_2}{\partial y}-\varepsilon^2\frac{\partial v_2}{\partial x}-p_2\,\mathrm{tg}\,\beta\right)\Big|_F; \tag{1.8}$$

$$\left[\alpha\left(\varepsilon^2\frac{\partial u_1}{\partial y}\,\mathrm{tg}\,\beta+\varepsilon^4\frac{\partial v_1}{\partial x}\,\mathrm{tg}\,\beta-2\varepsilon^2\frac{\partial v_1}{\partial y}\right)-p_1\right]\Big|_F=\left(\varepsilon^2\frac{\partial u_2}{\partial y}\mathrm{tg}\,\beta+\varepsilon^4\frac{\partial v_2}{\partial x}\,\mathrm{tg}\,\beta-2\varepsilon^2\frac{\partial v_2}{\partial y}-p_2\right)\Big|_F, \tag{1.9}$$

where $F = F(x)$ is the dimensionless profile of the surface of discontinuity; $\tan\beta = dF/dx$. We supplement the conservation conditions on the surface of discontinuity by the condition of continuity of the tangential component of velocity (adhesion of flows) and the local Michelson law:

$$(u_1 + \varepsilon^2 v_1\,\mathrm{tg}\,\beta)|_F = (u_2 + \varepsilon^2 v_2\,\mathrm{tg}\,\beta)|_F; \tag{1.10}$$

$$(-u_1\,\mathrm{tg}\,\beta + v_1)|_F = (1 + \varepsilon^2\,\mathrm{tg}^2\,\beta)^{1/2}. \tag{1.11}$$

The viscosity of the reaction product in the case of polymerization increases greatly (by four to seven orders of magnitude) compared to the viscosity of the initial mixture. Here, the dimensionless parameters α and Re_2 are considerably less than unity.

2. We will solve the problem in a zeroth approximation with respect to a small Reynolds number Re_2 and a small ratio of the transverse and longitudinal scales ε. Equations (1.1)-(1.6) and conditions (1.7)-(1.11) do not contain any terms on the order of ε, so the zeroth approximation will be valid to within terms of the order ε^2. With the same accuracy, Eqs. (1.2) and (1.5) and condition (1.9) lead to the conclusion that the pressure is constant

across the channel [p = const (y)], while the remaining equations and conditions take the form

$$\alpha\partial^2 u_1/\partial y^2 = dp/dx; \tag{2.1}$$

$$\partial^2 u_2/\partial y^2 = dp/dx; \tag{2.2}$$

$$\partial u_1/\partial x + \partial v_1/\partial y = 0; \tag{2.3}$$

$$\partial u_2/\partial x + \partial v_2/\partial y = 0; \tag{2.4}$$

$$u_1|_F = u_2|_F; \tag{2.5}$$

$$\alpha\partial u_1/\partial y|_F = \partial u_2/\partial y|_F; \tag{2.6}$$

$$v_1|_F = v_2|_F; \tag{2.7}$$

$$(-u_1 \operatorname{tg}\beta + v_1)|_F = 1. \tag{2.8}$$

It should be noted that the small parameter α in Eq. (2.1) is with the higher derivative and may introduce a singular perturbation into the solution. Thus, the terms containing the multiplier α cannot be deliberately discarded (see [16], for example).

We also augment the conditions on the surface of discontinuity with the condition of symmetry of the flow relative to the x axis, conditions of adhesion and impermeability on the wall, and an integral condition of conservation of flow rate over an arbitrary cross section of the channel:

$$\partial u_1/\partial y|_{y=0} = 0; \tag{2.9}$$

$$u_2|_{y=1} = v_2|_{y=1} = 0; \tag{2.10}$$

$$\int_0^F u_1 dy + \int_F^1 u_2 dy = 1. \tag{2.11}$$

Equations (2.1)-(2.4) and conditions (2.5)-(2.11) determine the problem being examined. Integrating (2.1) and (2.2) over y and (2.3) and (2.4) over x and inserting the results into Eqs. (2.5)-(2.11), after performing some transformations we obtain

$$x = x_0(F) = \int_{|F|}^1 \frac{3}{2}\frac{(1-\xi)(1+\xi+2\gamma\xi^2)}{(1+\gamma\xi^3)^2}d\xi = \frac{2-3|F|+|F|^3}{2(1+\gamma|F|^3)}; \tag{2.12}$$

$$u_1 = 3[(1+\gamma F^2) - (1+\gamma)y^2]/2(1+\gamma|F|^3); \tag{2.13}$$

$$u_2 = 3(1-y^2)/2(1+\gamma|F|^3); \tag{2.14}$$

$$v_1 = \gamma|F|y[(2-3|F| - \gamma|F|^3) + (1+\gamma)|F|y^2]/(1-|F|)(1+|F|+2\gamma F^2); \tag{2.15}$$

$$v_2 = \gamma F^2(1-y^2)(2+y)/(1-|F|)(1+|F|+2\gamma F^2); \tag{2.16}$$

$$\Delta p = \frac{9}{2}\int_{|F|}^1 \frac{(1-\xi)(1+\xi+2\gamma\xi^2)}{(1+\gamma\xi^3)^3}d\xi, \tag{2.17}$$

where $\gamma = \alpha^{-1} - 1$; Δp is the pressure gradient between the reference section, in which the surface of discontinuity adjoins the walls of the channel, and the current section; the explicit expression for the integral (2.17) is awkward and is omitted here. Equations (2.12)-(2.17) give the explicit dependence of the hydrodynamic fields on the coordinate y and the parametric dependence on x. Figure 1a shows profiles of the surface of discontinuity with $\alpha = 1$, 10^{-1}, 10^{-2}, and 10^{-6} (lines 1-4). It follows from (2.12) that

$$x_0(1) = 0,\ x_0(0) = 1,\quad x_0'(1) = 0,\ x_0'(0) = -3/2,\ \lim_{\alpha\to 0, F\neq 0} x_0(F) = 0.$$

In the reference section, the surface of discontinuity is turned so that it is perpendicular to the channel walls. The coordinate of the apex and the angle of inclination of the surface of discontinuity to the axis at the apex are independent of α. With a decrease in α to zero, the function $x_0(F)$ on the intervals $[-1, 0)$ and $(0, 1]$ uniformly converges to zero and the surface of discontinuity is distorted into a T-shape. Such behavior of the solution means that the small parameter α introduces a singular perturbation into it [16]. Here, the condition adopted earlier regarding the uniform smallness of the angle β over the entire flow region ceases to be satisfied. The angle β remains small until the surface of discontinuity is located in the neighborhood of the axis. When the surface is away from the axis and near the reference section, it becomes distorted and is oriented almost normal to the axis.

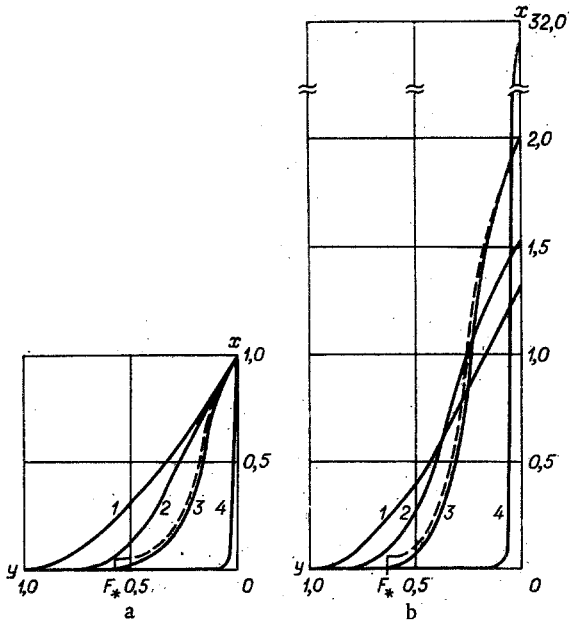

Fig. 1

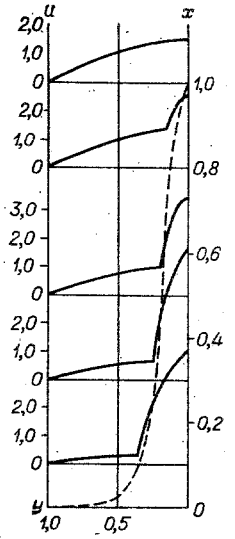

Fig. 2

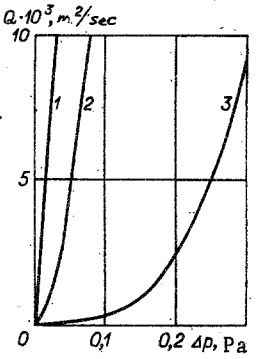

Fig. 3

Figure 2 shows profiles of the longitudinal component of velocity in different sections of the channel with $\alpha = 0.01$ (the dashed line represents the surface of discontinuity). It is apparent that the peripheral part of the channel is filled with a low-fluidity reaction product and that the central part is filled by a stream of unreacted, faster-moving material. The reaction product is entrained by the axial stream with an increase in the coordinate x, and the velocity of the product increases. Here, the velocity of the initial mixture on the axis initially increases due to a decrease in the channel cross section and then decreases to its former value due to the transfer of momentum to the reaction product.

The transverse component of velocity calculated from Eqs. (2.15) and (2.16) is small compared to the longitudinal component in the region of small values of the angle β. Near the reference section, where the angle β increases sharply, this scale relation is disturbed [Eqs. (2.15) and (2.16) have a singularity at $x = 0$ $(F - 1)$].

At flow rates which are fairly large for the exit of the surface of discontinuity from the channel (the "breakdown" regime $Q \geqslant 2LV$, where Q is the volumetric flow rate and L is the length of the channel), the flow-rate–discharge characteristic can be calculated from Eqs. (2.12), (2.17). At lower flow rates, the surface of discontinuity is localized inside the channel and the characteristic takes the form

$$\Delta p = \frac{9}{2}\int_0^1 \frac{(1-\xi)(1+\xi+2\gamma\xi^2)}{(1+\gamma\xi^3)^3}d\xi + 3(l-1),$$

where $l = L/X = 2LV/Q$ is the dimensionless length of the channel.

Figure 3 shows the dimensional flow-rate–discharge characteristics of a plane channel with $\alpha = 1.10^{-1}$ and 10^{-2} (lines 1-3, $Y = 7.5\cdot10^{-2}$ m, $V = 2.5\cdot10^{-4}$ m/sec, $\mu_1 = 5.7\cdot10^{-4}$ kg/m·sec, $\varepsilon = 0.1$). The volume of the part of the channel occupied by the high-viscosity reaction product decreases with an increase in the flow rate, and the characteristic becomes steeper. It is significant that the flow-rate–discharge characteristics in this case are monotonic and that no hysteresis phenomena are observed [3].

3. As was shown in Part 2, the solution obtained with the assumption of uniform smallness of the angle β over the entire flow region becomes incorrect near the reference section. To find a steady-state solution valid over the entire flow region, it is necessary to drop this assumption. Taking the angle β to be arbitrary, we find that the equations of motion in dimensionless variables do not change and that the conditions on the surface of discontinuity take the form*

*In Eqs. (3.1)-(3.5), in contrast to (1.7)-(1.9), $\tan\beta$ is determined through dimensional variables F and x for convenience in evaluating the orders of magnitude of the quantities and, as before, it is on the order of unity.

$$(u_1 \operatorname{tg}\beta - \varepsilon v_1)|_F = (u_2 \operatorname{tg}\beta - \varepsilon v_2)|_F; \tag{3.1}$$

$$\left[\alpha\left(2\varepsilon^2\frac{\partial u_1}{\partial x}\operatorname{tg}\beta - \varepsilon\frac{\partial u_1}{\partial y} - \varepsilon^3\frac{\partial v_1}{\partial x}\right) - p_1\operatorname{tg}\beta\right]\Big|_F = \left(2\varepsilon^2\frac{\partial u_2}{\partial x}\operatorname{tg}\beta - \varepsilon\frac{\partial u_2}{\partial y} - \varepsilon^3\frac{\partial v_2}{\partial x} - p_2\operatorname{tg}\beta\right)\Big|_F; \tag{3.2}$$

$$\left[\alpha\left(\varepsilon\frac{\partial u_1}{\partial y}\operatorname{tg}\beta + \varepsilon^3\frac{\partial v_1}{\partial x}\operatorname{tg}\beta - 2\varepsilon^2\frac{\partial v_1}{\partial y}\right) - p_1\right]\Big|_F = \left(\varepsilon\frac{\partial u_2}{\partial y}\operatorname{tg}\beta + \varepsilon^3\frac{\partial v_2}{\partial x}\operatorname{tg}\beta - 2\varepsilon^2\frac{\partial v_2}{\partial y} - p_2\right)\Big|_F; \tag{3.3}$$

$$(u_1 + \varepsilon v_1 \operatorname{tg}\beta)|_F = (u_2 + \varepsilon v_2 \operatorname{tg}\beta)|_F; \tag{3.4}$$

$$(-u_1 \operatorname{tg}\beta + \varepsilon v_1)|_F = \varepsilon(1 + \operatorname{tg}^2\beta)^{1/2}. \tag{3.5}$$

Conditions (3.1) and (3.4) are equivalent to (2.5) and (2.7). Discarding terms on the order of ε^2 in (3.2) and (3.3), after several transformations we obtain

$$\alpha\partial u_1/\partial y|_F = \partial u_2/\partial y|_F + O(\varepsilon); \tag{3.6}$$

$$p_1|_F = p_2|_F + O(\varepsilon^2). \tag{3.7}$$

In the zeroth approximation with respect to ε, the last conditions coincide, respectively, with (2.6) and the pressure continuity condition. However, in contrast to (2.6), condition (3.7) turns out to be valid only to within terms on the order of ε.

Condition (3.5) is quite different from (2.8), which corresponds to a small angle β. Here, we have the following for the surface of discontinuity:

$$x = x_1(F) = 1 - \int_0^{|F|} (f^2(\xi) - \varepsilon^2)^{1/2} d\xi, \tag{3.8}$$

where $f(\xi)$ is the integrand in (2.12). The limits of integration in (3.8) were calculated so that the coordinates of the apex of the surface of discontinuity calculated from this formula and from Eq. (2.12) would coincide. Equations (2.13)-(2.17) remain the same for the hydrodynamic fields.

The dashed lines in Fig. 1 show the profile of the surface of discontinuity calculated from (3.8) with $\alpha = 0.01$ and $\varepsilon = 0.1$. Away from the reference section the angle β is small, $f \gg \varepsilon$, and solution (3.8) turns out to be close to (2.12):

$$x = x_1(F) = x_0(F) - \frac{\varepsilon^2}{2}\int_0^{|F|}\frac{d\xi}{f(\xi)} + o(\varepsilon^2). \tag{3.9}$$

The value of f decreases and the last formula becomes inaccurate as the reference section is approached. With attainment of the critical section $F = F_*$, in which

$$f = f_* = \varepsilon, \tag{3.10}$$

the surface of discontinuity is turned so that it is normal to the axis $((dx_1/dF)|_{F_*} = 0)$, and solution (3.8) ceases to exist.

The disappearance of the steady-state solution at $F > F_*$ is connected with the incompatibility of conditions (2.10) on the channel walls and the Michelson law in the form (3.5), which corresponds to a similarity wave regime of propagation of the reaction. The solution obtained on the basis of the Michelson law should be quite nonsteady in this region, since at $F > F_*$ the velocity of the wave V becomes greater than the local flow velocity and the wave will penetrate upstream. To realize a steady-state regime, it is necessary for the reaction propagation regime to change sufficiently far from the channel axis: here, a "restraining zone" which stabilizes the reaction wave [14] should exist.

In the case of polymerization, $\alpha \ll \varepsilon$, and the asymptotic expression $F_* = (3\alpha/\varepsilon)^{1/4} + o(\alpha^{1/4})$ can be obtained for the root of Eq. (3.10). It follows from this expression that with sufficiently small α the region in which no steady-state solution exists covers most of the channel cross section.

4. Let us examine stationary flows in a cylindrical channel with similarity propagation of the reaction. The system of equations of motion in dimensionless variables takes the following form in the zeroth approximation with respect to small Re_2 and ε:

$$\alpha\frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial u_1}{\partial y}\right) = \frac{dp}{dx}, \quad \frac{1}{y}\frac{\partial}{\partial y}\left(y\frac{\partial u_2}{\partial y}\right) = \frac{dp}{dx},$$

$$\frac{\partial u_1}{\partial x} + \frac{1}{y}\frac{\partial}{\partial y}(yv_1) = 0, \quad \frac{\partial u_2}{\partial x} + \frac{1}{y}\frac{\partial}{\partial y}(yv_2) = 0,$$

$$\int_0^F u_1 y dy + \int_F^1 u_2 y dy = 1/2,$$

where x and y are the axial and radial coordinates; u and v are the axial and radial components of the velocity vector, having the same notation as the corresponding coordinates and velocity components in the case of a plane channel. The form of conditions (2.5)-(2.10) remains the same.

It can be shown such a formulation of the problem is equivalent to that proposed in [12], where equations of motion of a medium with variable viscosity were written for the entire flow region. However, the approach examined in the present work reveals the role of the small parameter α and makes it possible to solve the problem with arbitrary values of the angle β.

Performing the necessary calculations, we obtain the following for the sought variables

$$x = x_0(F) = \int_F^1 2\frac{(1-\xi^2)(1+\gamma\xi^2)}{(1+\gamma\xi^4)^2}d\xi; \tag{4.1}$$

$$u_1 = 2[(1+\gamma F^2) - (1+\gamma)y^2]/(1+\gamma F^4); \tag{4.2}$$

$$u_2 = 2(1-y^2)/(1+\gamma F^4); \tag{4.3}$$

$$v_1 = \gamma F y[(1-2F^2-\gamma F^4) + (1-\gamma)F^2y^2]/(1-F^2)(1 \tag{4.4}$$

$$+\gamma F^2); \tag{4.5}$$

$$v_2 = \gamma F^3(1-y^2)^2/y(1-F^2)(1+\gamma F^2);$$

$$\Delta p = 16\int_F^1 \frac{(1-\xi^2)(1+\gamma\xi^2)}{(1+\gamma\xi^4)^3}d\xi. \tag{4.6}$$

Profiles of the surface of discontinuity for different values of α are shown in Fig. 1b. As in the case of a plane channel, the small parameter α introduces a singular perturbation into the solution. This leads to sharp distortion and extension of the profile in the neighborhood of the axis (the values of α are the same as in Fig. 1a).

An important feature of the cylindrical channel is that with a decrease in α to zero, the coordinate of the apex of the surface of discontinuity increases without limit rather than remaining constant. Here, the axial stream of unreacted material is highly extended along the channel axis, which leads to "breakdown" of the tube reactor [1-4] at relatively low flow rates.

It should be noted that given an initial mixture and a product with the same velocities, $\alpha = 1$, and (4.1) takes the form $x = x_0(F) = (2/3)(F^3 - 3F + 2)$. This form of (4.1) coincides with the well-known solution for the shape of the flame from a bunsen burner [14].

The flow-rate–discharge characteristic of the cylindrical channel at $Q \geqslant \pi YLV/x_0(0)$ ("breakdown" regime) is given parametrically by Eqs. (4.1) and (4.6). At lower flow rates it takes the form

$$\Delta p = 16\int_0^1 \frac{(1-\xi^2)(1+\gamma\xi^2)}{(1+\gamma\xi^4)^3}d\xi + 8[l - x_0(0)],$$

where $l = L/X = \pi YLV/Q$ is the dimensionless length of the channel. The flow-rate–discharge characteristics coincide with those obtained in [12] and are qualitatively the same as the characteristics for a plane channel.

When the condition of uniform smallness of the angle β is dropped, we have the following for the profile of the surface of discontinuity

$$x = x_1(F) = x_0(0) - \int_0^F (f^2(\xi) - \varepsilon^2)^{1/2}d\xi,$$

where $f(\xi)$ is the integrand in (4.1). Equations (3.9) and (3.10) have the same form.

The asymptotic expression for the root of Eq. (3.10) in the case of a large increase in viscosity on the surface of discontinuity $F_* = (2\alpha/\varepsilon)^{1/6} + o(\alpha^{1/6})$. In the case of equal viscosities, we obtain the expression $F_* = (1 - \varepsilon/2)^{1/2}$ for the root. This expression coincides with the analogous formula obtained from the solution for the bunsen burner [14].

LITERATURE CITED

1. S. A. Vol'fson and N. S. Enikolopyan, Calculations of Highly Efficient Polymerization Processes [in Russian], Khimiya, Moscow (1980).
2. R. Sala, F. Valz-Gris, and L. Zanderighi, "A fluid-dynamic study of a continuous polymerization reactor," Chem. Eng. Sci., 29, No. 11 (1974).
3. S. A. Bostandzhiyan, V. I. Boyarchenko, et al., "Low-temperature polymerization regimes in a continuous reactor," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1979).
4. A. A. Butakov and A. M. Zanin, "Experimental study of the laws of an exothermic reaction with a reactant of variable viscosity in a tube reactor," Fiz. Goreniya Vzryva, 14, No. 5 (1978).
5. D. A. Vaganov, "Some two-dimensional effects in the flow of a reacting liquid with properties which change with the degree of conversion," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1977).
6. D. A. Vaganov, "Stationary flow of a reacting liquid with properties which change with the degree of conversion," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1984).
7. N. M. Chechilo, R. Ya. Khvilivitskii, and N. S. Enikolopyan, "Phenomena regarding the propagation of a polymerization reaction," Dokl. Akad. Nauk SSSR, 204, No. 5 (1972).
8. B. B. Khanukaev, M. A. Kozhushner, and N. S. Enikolopyan, "Theory of propagation of a polymerization front," Dokl. Akad. Nauk SSSR, 214, No. 3 (1974).
9. S. P. Davtyan, N. F. Surkov, et al., "Kinetics of radical polymerization under conditions of propagation of the reaction front with allowance for the gel effect," Dokl. Akad. Nauk SSSR, 232, No. 2 (1977).
10. G. V. Zhizhin, "Structure of frontal polymerization waves," Dokl. Akad. Nauk SSSR, 263, No. 6 (1982).
11. G. V. Zhizhin, "Calculation of the structure of a frontal polymerization wave," Kinet. Katal., 25, No. 2 (1984).
12. D. A. Vaganov and P. V. Zhirkov, "Hydrodynamic laws of frontal polymerization in a tube reactor," in: Summary of Documents of the Eighth All-Union Conference on Chemical Reactors [in Russian], Vol. 2, Chimkent (1983).
13. G. G. Chernyi, "Laminar motions of gas and liquid in a boundary layer with a surface of discontinuity," Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk, No. 12 (1954).
14. Ya. B. Zel'dovich, G. I. Barenbaltt, et al., Mathematical Theory of Combustion and Explosion [in Russian], Nauka, Moscow (1980).
15. H. Schlichting and K. Schlichting, Boundary Layer Theory, 6th ed., McGraw-Hill, New York (1968).
16. S. A. Lomov, Introduction to the Theory of Singular Perturbations [in Russian], Nauka, Moscow (1981).